Transient states depend on the dimensionality of thermal convection

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Abstract

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O. Introduction

A practical problem concerning the direct numerical simulations (DNS) of thermal convection is the amount of time it takes for the flow to reach the steady state. Without reliable idea of how long the transient state lasts, one cannot be certain about the numerical results obtained. In (1), we studied this problem in some detail for 2D convection. Here, we will combine these results with those in 3D convection—and, to complete the narrative, also consider convection in 1D and 4D settings, being fully aware of the esoteric nature of the last two. We show that some general principles can be learnt by considering these results in totality.

1. 1D convection

A moment's thought shows that, when the bottom of a 1D system is heated and its top cooled, heat can be conveyed from the bottom to the top only by conduction in incompressible fluids; there can never be any convection at any Rayleigh or Prandtl numbers. One

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can readily interpret this result to mean that the transient state to convection is infinitely long. In compressible cases, pressure perturbations can travel between the two plates. Even here, there can be no bodily movement of the fluid—that is, there is no convection. The oscillatory character of the pressure fluctuations has a dependence on the Prandtl number but the basic result is the same. A supporting statement can be brought to bear on this result from stability considerations.

2. 2D convection

In Pandey & Sreenivasan (1) we showed, using data between Ra = 107 and Ra12, that the transient time varies from Ra0.53 to Ra0.7, approximately consistent with the lower bound theory of Lindborg (2), but the coefficient depended on the Prandtl number. More recent considerations suggest that all the data on transient states, with Pr varying from 0.021 to 1, follow a nearly linear relation with the Reynolds number of convection, based on the free fall velocity. This strong dependence of the transient times on the Reynolds number occurs because the root-mean-square velocity of 2D convection grows with Ra faster than the free-fall velocity, which itself may be a consequence of inverse cascade..

We note that the data provided are for ab initio calculations (i.e., convection begun from slightly perturbed conduction states). At the steady state, the non-dimensional kinetic energy is higher for higher Ra, so there will again be a transient period when simulation is started from the lower Ra. However, the transient time will be shorter than if a simulation starts from the conduction state.

3. 3D convection

This is the practically relevant case. The collection of evidence from the considerable body of work on this aspect suggests that the initial perturbations grow exponentially but saturate to an asymptotic state that is effectively constant (despite somewhat oscillatory behavior). The body of our work suggests that the final state is reached after 30-50 free fall times, nearly independent of the Rayleigh and Prandtl numbers—thus the Reynolds number. We cannot exclude weak dependencies (mainly because we have not yet developed reliable quantitative measures), and the time required for the steady state to be reached could thus be higher at higher Ra.

4. 4D convection

Here, we consider convection in four spatial dimensions and time. Since 4D convection never appears in the universe, a sentence may be necessary to justify the effort. First, it will enable to view the problem of transients from a broader perspective. Second, motivated by the success of papers such as (3) on critical phenomena, there has been a thought in the turbulence literature (for a ”relatively recent” article, see (4)) that turbulence in 4D may be free of intermittency corrections; corresponding expectations on heat transport can be built up for 4D convection. Due to the significant computational demand of these simulations, simulations in 4D convection are currently restricted to relatively low Rayleigh numbers and simplified boundary conditions. Results up to Ra=10ˆ6 reveal that the transition to a steady state is qualitatively similar to the 3D case. We are currently exploring higher Rayleigh numbers to verify if the transient duration becomes independent of the Reynolds number in the turbulent regime, as in the 3D case.

5. Tentative conclusion

The results can be expressed very qualitatively as

Transient time =

where

and b = 40 for D = 3 and 4 and = 10-2 for D = 2.

References

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