Stochastic Lorenz dynamics and wind reversals in Rayleigh-Bénard Convection

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Abstract

The Lorenz equations are a severe Galerkin-truncation of the Oberbeck-Boussinesq (OB) equations describing Rayleigh-Bénard convection (RBC). Here we examine the mathematical connections between the chaotic lobe-switching behavior of a stochastic form of the Lorenz equations, that model the interaction between the thermal boundary layers and the core circulation, and the mean wind reversals in the experiments of Sreenivasan et al. Long-time numerical simulations of these stochastic equations, not easily accessible with the OB equations, yield a probability distribution for lobe inter-switch timings that exhibits non-Gaussian, multifractal behavior. In the Gaussian frequency range the simulations mirror the laboratory measurements and the classical Hurst exponent and quadratic variation show Brownian second-moment statistics. Further scrutiny reveals a non-linear cumulant generating function, or moment-exponent function, and thus multifractality. A simple generalized two-scale Cantor-cascade analysis reproduces these properties, showing that multiplicative intermittency, characteristic of turbulence, strongly influences the statistics. This demonstrates that this stochastic Lorenz system is a faithful, low-dimensional surrogate for mean-wind reversals in RBC.

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